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Curvaton reheating: An application to braneworld inflation

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The curvaton was introduced recently as a distinct inflationary mechanism for generating adiabatic density perturbations. Implicit in that scenario is that the curvaton offers a new mechanism for reheating after inflation, as it is a form of energy density not diluted by the inflationary expansion. We consider curvaton reheating in the context of a braneworld inflation model, *steep inflation*, which features a novel use of the braneworld to give a new mechanism for ending inflation. The original steep inflation model featured reheating by gravitational particle production, but the inefficiency of that process causes observational difficulties. We demonstrate here that the phenomenology of steep inflation is much improved by curvaton reheating.

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I. INTRODUCTION

The curvaton [1,2] is an interesting new proposal for explaining the observed large-scale adiabatic density perturbations in the context of inflation. Perturbations are acquired by a field other than the inflaton during inflation, whose energy density is subdominant during inflation but which comes to dominate sometime afterward, at which point its initial isocurvature perturbations convert to adiabatic [1,3] (see also [4]). Decay of the curvaton to conventional matter locks the perturbations in as adiabatic, although there can be complex phenomenology if some species have already decoupled from the thermal bath by the time this happens [2,5].

An aspect of this process which has been implicit in most papers is that this process also offers a new mechanism of reheating. The first detailed discussion of this aspect appeared recently in Ref. [6] by Feng and Li, who named it *curvaton reheating*. The curvaton is a type of matter whose energy density is not diluted during inflation, and some or all of the present material in the Universe may have survived inflation via the curvaton. This reheating mechanism complements the other two known mechanisms, the standard one being decay of the inflaton energy density into conventional matter and the rarely used alternative being gravitational particle production at the end of inflation [7].

In this paper we will explore an application of curvaton reheating to a particular model of braneworld inflation known as *steep inflation* [8]. This model brings the inflationary epoch to an end in a novel way. The potential chosen (for example a steep exponential) is so steep that it would not drive inflation in the standard cosmology, but it may do so in the presence of high-energy braneworld corrections to the Friedmann equation, for example as found in the Randall-Sundrum type II scenario. As the energy density decreases, those corrections become unimportant, and the scalar field makes a transition into a kinetic energy dominated regime bringing inflation to an end. As the inflaton survives this process without decay, an alternative reheating mechanism is required.

In Ref. [8], it was proposed that reheating took place via gravitational particle production [7,9]. However this brings problems to the scenario. Because of the inefficiency of the process, there is a long kinetic energy dominated regime, which allows short-wavelength gravitational waves to reach excessive amplitudes [10]. There are likely also to be problems with the low reheat temperature achieved, which is barely sufficient to allow standard nucleosynthesis to proceed. The scenario also suffers from a separate problem, which is that it tends to predict perturbation spectra which are too far from scale invariance, and too high an amplitude of long-wavelength gravitational waves; for the exponential potential the predicted spectra are in conflict with the most recent observations.

In this paper, we propose that implementation of curvaton reheating in place of gravitational particle production can redress those difficulties. Curvaton reheating can be much more efficient, allowing a high reheat temperature and preventing short-wavelength gravitational wave dominance. Further, adiabatic perturbations produced via the curvaton may be closer to scale invariance, and in particular the contribution of gravitational waves to the microwave anisotropies may be considerably suppressed. Two related papers appeared as we were completing this work; Ref. [6] discusses curvaton reheating in a quintessential inflation model [11] in the standard cosmology, while Ref. [12] considers adding the curvaton to the steep inflation model, but focusing on particle physics model building aspects rather than observational phenomenology and constraints.

A summary of this paper is as follows. In Sec. II, we review the main features of steep inflation in the braneworld scenario. We present only the main results of these kinds of models, including the production of primordial gravitational waves. In Sec. III, we study the evolution of a curvaton field and its perturbations in steep inflation, and we focus our attention on all the constraints to be applied upon the free parameters of the model. We consider separately the cases that arise depending on whether the curvaton field dominates after or before it decays into radiation. In Sec. IV we put all constraints together and analyze the parameter space of the

model of curvaton field + steep inflation which satisfies all the requirements. In the final section we draw some conclusions and indicate topics to be investigated in future publications.

II. THE STEEP INFLATION MODEL

In this section we give a brief description of an inflationary model in the braneworld scenario, assuming that the inflaton field is a minimally coupled real scalar field ϕ endowed with an exponential potential of the form

$$V(\phi) = V_0 e^{-\alpha \sqrt{\kappa_0} \phi}, \quad (1)$$

where V_0 , α are free parameters and $\kappa_0 = 8\pi G = 8\pi m_{\text{Pl}}^{-2}$ with m_{Pl} the Planck mass. More details of the model can be found in Refs. [8,10]. A somewhat related idea has been discussed in Ref. [13].

A. Inflationary dynamics

In the $(4+1)$ -dimensional brane scenario inspired by the Randall-Sundrum model, the standard Friedman equation is modified to

$$H^2 = \frac{\kappa_0}{3} \rho \left(1 + \frac{\rho}{2\lambda_b} \right), \quad (2)$$

where λ_b is the brane tension and, taking the inflaton field to be the dominant component, $\rho = \dot{\phi}^2/2 + V(\phi)$. The scalar field is confined to the brane so that its equation of motion is the usual Klein-Gordon one

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad (3)$$

where overdots denote temporal derivatives.

For large energies $\rho \gg \lambda_b$, the inflaton field experiences an increased friction as it rolls down its potential, and then one can expect inflation to occur even for steep potentials (i.e. even for $\alpha \gg 1$). This can be seen from the slow-roll parameters for the exponential potential [14], which for large energies read

$$\epsilon = \eta = 2\alpha^2 \frac{\lambda_b}{V}, \quad (4)$$

and thus inflation can occur whenever $V > 2\alpha^2 \lambda_b$.

The equations of motion can be solved exactly in the slow-roll approximation, from which we find the following expressions for the scale factor a , the Hubble factor H and the inflaton field ϕ , respectively:

$$\ln \frac{a}{a_i} = (\mathcal{N} + 1) - e^{-H_f(t-t_f)}, \quad (5)$$

$$H = H_f e^{-H_f(t-t_f)}, \quad (6)$$

$$\alpha \sqrt{\kappa_0} \phi(t) = H_f(t-t_f) - \ln \frac{V_f}{V_0}. \quad (7)$$

Here “i” (“f”) indicates the initial (final) value of the different quantities during inflation, and \mathcal{N} is the number of e -foldings as can be seen from Eq. (5). Observe that, in the braneworld scenario, the slow-roll approximation for the exponential potential is a much better approximation than in standard cosmology.

Inflation ends by violation of the slow-roll conditions, when $\epsilon \approx \eta \sim 1$, and so the value of the potential energy at the end of inflation is $V_f \approx 2\alpha^2 \lambda_b$. Using Eqs. (5)–(7), we find that the initial and final values of the Hubble factor and the potential energy, respectively, are related through

$$\frac{H_i}{H_f} = \frac{V_i}{V_f} \approx \mathcal{N} + 1, \quad (8)$$

where

$$H_f = \alpha^2 \sqrt{\frac{2\lambda_b \kappa_0}{3}}. \quad (9)$$

So far the model has three free parameters, but only two of them are relevant for inflation, the brane tension λ_b and the self-interaction of the inflaton field α .

The amplitude of scalar density perturbations can be calculated for the exponential potential, and it is given by [8,10]

$$A_s^2 \approx \frac{8}{75} \frac{V_i^4}{\alpha^2 \lambda_b m_{\text{Pl}}^4} = \frac{128}{75} \alpha^6 (\mathcal{N} + 1)^4 \frac{\lambda_b}{m_{\text{Pl}}^4}. \quad (10)$$

In the usual scenario of steep inflation, the values of the inflation parameters can be adjusted so that we get the observed value $A_s^2 \approx 4 \times 10^{-10}$. In particular, we can determine the value of the brane tension in terms of α as

$$\lambda_b \approx \frac{2.3 \times 10^{-10}}{\alpha^6} \frac{m_{\text{Pl}}^4}{(\mathcal{N} + 1)^4}. \quad (11)$$

Notice that any deviation of λ_b from the value obtained in Eq. (11) would imply either a larger or smaller amplitude for the scalar spectrum.

As was pointed out in Ref. [10], the number of e -foldings can be unambiguously determined for the exponential potential, for which we obtain $\mathcal{N} \approx 70$. This result is based on Eq. (10) and is independent of the value of A_s^2 .

B. After inflation

Soon after inflation ends, the brane term becomes unimportant and we recover the standard Friedman equation. Hence, the friction on the inflaton field diminishes and the inflaton energy is dominated by the kinetic term and the scalar energy density falls as stiff matter, $\rho_\phi = \rho_\phi^{(\text{kin})} (a_{\text{kin}}/a)^6$. The epoch that follows is called the “kinetic epoch” or “kination” [15], and we shall use “kin” to label the value of the different quantities at the beginning of the kinetic epoch.

Since we are now working within standard cosmology (at energies $\rho < \lambda_b$), the Hubble factor evolves as

$$H = H_{\text{kin}} \left(\frac{a_{\text{kin}}}{a} \right)^3, \quad H_{\text{kin}}^2 \simeq \frac{\kappa_0}{3} \rho_{\phi}^{(\text{kin})}. \quad (12)$$

The kinetic regime does not commence immediately after the end of inflation, but the values of the Hubble parameter at the end of inflation and at the beginning of the kinetic regime are related by the formula [10]

$$\frac{H_{\text{kin}}}{H_{\text{f}}} = 0.085 - \frac{0.688}{\alpha^2}. \quad (13)$$

A small amount of radiation has been produced quantum-mechanically during inflation with an energy density $\rho_{\text{r}} \sim 0.01 g_p H_{\text{f}}^4$, where $g_p = 10\text{--}100$ is the number of different species created from the vacuum. If the inflaton potential does not decay just after inflation, then this is the only mechanism to reheat the Universe.

With the density of scalar matter falling more rapidly than that of radiation, the Universe continues evolving according to Eq. (12) until the small amount of radiation comes to dominate. Since the ratio of scalar matter to radiation at the end of inflation is large, $(\rho_{\phi}/\rho_{\text{r}})_{\text{f}} \gg 1$, radiation comes to dominate the expansion of the Universe only after a very long kinetic epoch. This is troublesome since the energy density of gravitational waves produced quantum-mechanically can exceed the nucleosynthesis constraints, as shown in Ref. [10].

In braneworld inflation, the amplitude of gravitational waves h_{GW}^2 is enhanced with respect to the standard scenario, and its value at the end of inflation is given by

$$h_{\text{GW}}^2 \simeq 3 \alpha^2 (\mathcal{N} + 1) \frac{H_{\text{i}}^2}{m_{\text{Pl}}^2}. \quad (14)$$

Gravitational waves behave as massless scalar fields and then their amplitude remains constant during inflation. During the kinetic epoch, the energy density of gravitational waves evolves as [10]

$$\rho_{\text{g}} = \frac{32}{3\pi} h_{\text{GW}}^2 \rho_{\phi} \left(\frac{a}{a_{\text{kin}}} \right)^2. \quad (15)$$

From this, the energy density of gravitational waves at the time of scalar stiff matter–radiation equality ($\rho_{\phi} = \rho_{\text{r}}$) is

$$\left. \frac{\rho_{\text{g}}}{\rho_{\text{r}}} \right|_{a=a_{\text{eq}}} = \frac{64}{3\pi} h_{\text{GW}}^2 \left(\frac{a_{\text{eq}}}{a_{\text{kin}}} \right)^2 \simeq \frac{64}{3\pi} h_{\text{GW}}^2 \frac{\rho_{\phi}}{\rho_{\text{r}}} \Big|_{\text{f}} \gg 1. \quad (16)$$

Thus, the energy density of gravitational waves on *short wavelength scales* [10] exceeds the contribution of radiation. It happens then that gravitational waves come to dominate the stiff scalar matter well before radiation, which in fact never dominates, and then nucleosynthesis constraints are not satisfied at all. Therefore, the original steep inflation model seems to be ruled out [10].

However, the production of gravitational waves can be kept under control if the kinetic epoch is shortened. The latter can be done either by a sufficiently early decay of the

inflaton field or by having a post-inflationary equation of state softer than that of stiff matter. The latter picture is quite appealing since the surviving inflaton field could become part of the dark matter at late times [10,16,17].

III. THE CURVATON FIELD

In this section we explore the possibility that the inflaton field is not responsible for providing the primordial fluctuations required for the formation of structure in the late Universe, as in the case where the value of the brane tension is lower than that in Eq. (11). Instead, the primordial fluctuations are generated by a second scalar field σ , usually called the *curvaton field*, through initially isocurvature perturbations [1]. For simplicity, we assume here that the curvaton field has a quadratic scalar potential $U(\sigma) = m^2 \sigma^2/2$, and hence obeys the Klein-Gordon equation

$$\ddot{\sigma} + 3H\dot{\sigma} + m^2\sigma = 0, \quad (17)$$

where m is the curvaton mass.

We will now follow the evolution of the curvaton field through different stages and find the constraints upon the free parameters of the model in order to have a viable curvaton scenario together with steep inflation. In doing this, we shall follow a similar procedure to that in Ref. [18]. We begin with a brief overview of the dynamics.

First of all, it is assumed that the curvaton field coexists with the inflaton field during inflation, during which the inflaton energy density is the dominant component. The curvaton field should survive the rapid expansion of the Universe, and for that it has to be effectively massless. As we shall see below, this condition is translated into a constraint on the curvaton mass m . If this constraint is satisfied, the curvaton field will remain constant at its initial value.

The following stage is that when the curvaton field becomes effectively massive, and this will generally happen during the kinetic epoch. In order to prevent a period of curvaton-driven inflation, the Universe must remain inflaton dominated until this time. The latter condition imposes a constraint on the initial values the curvaton field can take, σ_{i} . Once effectively massive, the curvaton field oscillates around the minimum of its potential and its energy density evolves as non-relativistic matter.

The final stage is that of curvaton decay into radiation, and the standard big bang cosmology is recovered afterward. In the general case, curvaton decay should occur before nucleosynthesis, but other constraints arise depending on epoch of the decay, governed by the decay parameter Γ_{σ} . There are two scenarios to be considered, depending whether the curvaton field decays before or after it becomes the dominant component of the Universe. Whatever the case, the energy from gravitational waves should be maintained under control, which will actually become an extra constraint involving different parameters of the model. Also, we calculate the curvaton perturbations for each case which will help us to reduce the number of free parameters. At this point, the Gaussianity conditions on the perturbations will be taken into account too.

The introduction of the curvaton field adds three free parameters to the steep inflation model: the curvaton mass m , the initial value of the curvaton field σ_i and the curvaton decay parameter Γ_σ . However, a successful model needs to satisfy the conditions outlined above which are strong enough to restrict the allowed curvaton parameters.

A. Inflation

We first study the evolution of the curvaton field during braneworld inflation with an exponential inflaton potential as described in Sec. II. We change the independent variable to $z \equiv 3H/H_f$, for which the equation of motion of the curvaton field Eq. (17) reads

$$z^2 \sigma'' + z(1-z) \sigma' + \frac{m^2}{H_f^2} \sigma = 0, \quad (18)$$

where primes denote derivatives with respect to z .

Assuming that the curvaton field is effectively massless during inflation $m \ll H_f$, the last term in Eq. (18) is small compared to the others, so that the curvaton equation of motion is

$$z^2 \sigma'' + z(1-z) \sigma' \simeq 0, \quad (19)$$

whose exact solutions are

$$\sigma(z) = \sigma_i + \sigma'_i z_i e^{-z_i} [\text{Ei}(z) - \text{Ei}(z_i)], \quad (20)$$

$$\sigma' = \sigma'_i \frac{z_i}{z} \exp(z - z_i). \quad (21)$$

Here, σ_i and σ'_i are the initial values of the field and its derivative, and $\text{Ei}(z)$ is the exponential integral. In the case that $\mathcal{N} = 70$, so that z runs in the range $z_i = 213 \rightarrow z_f = 3$ [see Eq. (8)], the ratio of the final value of the curvaton field to its initial value is

$$\frac{\sigma_f}{\sigma_i} = 1 - 1.005 \frac{\sigma'_i}{\sigma_i}. \quad (22)$$

We see that the final value σ_f depends on the initial conditions. The interesting case is that in which the curvaton energy is equally distributed into kinetic and potential forms at the beginning of inflation, so that we can take $\dot{\sigma}_i^2 \sim m^2 \sigma_i^2$. Thus, from $\dot{\sigma}_i = H_f z_i \sigma'_i$ we obtain

$$\frac{\sigma'_i(z)}{\sigma_i(z)} \sim \frac{m}{H_f z_i} = \frac{1}{3(\mathcal{N}+1)} \frac{m}{H_f}. \quad (23)$$

Therefore, the condition $m \ll H_f$ makes the curvaton field remain constant under general initial conditions, and we can write $\sigma_f \simeq \sigma_i$. Notice also that $\dot{\sigma}_f = 0$.

B. Kinetic epoch

As we anticipated at the beginning of this section, the dynamics of the curvaton field after inflation are different from the scenarios considered before in the literature. This

difference is mainly because the curvaton field coexists with the surviving inflaton field, whose energy density is by far the dominant one at the end of inflation. The consequences of this will be explored in this section and our immediate concern will be to work out the corresponding gravitational wave constraint.

According to our hypothesis, it is during the kinetic epoch that the curvaton field becomes effectively massive. This happens at a time when $m \simeq H$, from which we obtain [see Eq. (12)]

$$\frac{m}{H_{\text{kin}}} = \frac{a_{\text{kin}}^3}{a_m^3}, \quad (24)$$

where “m” labels the quantities at the time when the curvaton becomes massive. Up to this point, the curvaton field remained effectively massless and we can consider that $\sigma_m \simeq \sigma_i$.

In order to prevent a period of curvaton-driven inflation, the Universe must still be dominated by the scalar stiff matter at this point. Hence, using Eqs. (12) and (24), we arrive at the restriction

$$\frac{m^2 \sigma_i^2}{2\rho_\phi^{(m)}} = \frac{4\pi}{3} \frac{m^2 \sigma_i^2}{m_{\text{pl}}^2 m^2} \ll 1 \Rightarrow \sigma_i^2 \ll \frac{3}{4\pi} m_{\text{pl}}^2. \quad (25)$$

Equation (25) is sufficient to achieve subdominance at the time when the curvaton field becomes massive. But the curvaton energy should also be subdominant at the end of inflation, which implies

$$\frac{U_f}{V_f} = \frac{m^2 \sigma_i^2}{4\alpha^2 \lambda_b} \ll \frac{3m^2 m_{\text{pl}}^2}{16\pi \alpha^2 \lambda_b} = \alpha^2 \frac{m^2}{H_f^2}, \quad (26)$$

where we have used Eqs. (9) and (25). Thus, the curvaton mass should obey the stronger constraint

$$m \ll \alpha^{-1} H_f. \quad (27)$$

After the curvaton field becomes effectively massive, its energy decays as non-relativistic matter in the form

$$\rho_\sigma = \frac{m^2 \sigma_i^2}{2} \frac{a_m^3}{a^3}. \quad (28)$$

C. Curvaton domination

We calculate here the constraints to be applied on the curvaton model for the case in which the curvaton field comes to dominate the cosmic expansion, dealing first with the gravitational wave constraint. Since the curvaton energy redshifts as in Eq. (28), we find at the time of stiff scalar matter–curvaton matter equality ($\rho_\sigma = \rho_\phi$) that¹

¹Note that the label a_{eq} has different meanings in Eqs. (16), (29) and (35).

$$\left. \frac{\rho_\sigma}{\rho_\phi} \right|_{a=a_{\text{eq}}} = \frac{4\pi}{3} \frac{m^2 \sigma_i^2}{m_{\text{Pl}}^2 H_{\text{kin}}^2} \frac{a_{\text{m}}^3}{a_{\text{kin}}^3} \frac{a_{\text{eq}}^3}{a_{\text{kin}}^3} = 1. \quad (29)$$

Using Eqs. (24) and (29) in Eq. (15), we can calculate the energy density of gravitational waves at this time, which should obey the constraint

$$\left. \frac{\rho_g}{\rho_\sigma} \right|_{a=a_{\text{eq}}} = \frac{64}{3\pi} h_{\text{GW}}^2 \left(\frac{3}{4\pi} \frac{m_{\text{Pl}}^2}{\sigma_i^2} \frac{H_{\text{kin}}}{m} \right)^{2/3} \ll 1. \quad (30)$$

To finish with, we derive the conditions to be applied on the decay parameter Γ_σ . On the one hand, we require the curvaton field to decay before nucleosynthesis, and then $H_{\text{nuc}} = 10^{-40} m_{\text{Pl}} < \Gamma_\sigma$. On the other hand, we also require the curvaton to decay after domination, and then $\Gamma_\sigma < H_{\text{eq}}$, where H_{eq} is found by using Eqs. (12), (24) and (29):

$$H_{\text{eq}} = H_{\text{kin}} \frac{a_{\text{kin}}^3}{a_{\text{eq}}^3} = \frac{4\pi}{3} \frac{\sigma_i^2}{m_{\text{Pl}}^2} m. \quad (31)$$

Hence, the constraint upon the decay parameter is

$$10^{-40} m_{\text{Pl}} < \Gamma_\sigma < \frac{4\pi}{3} \frac{\sigma_i^2}{m_{\text{Pl}}^2} m. \quad (32)$$

D. Curvaton decay before domination

We now turn our attention to the case in which the curvaton field decays before it dominates the cosmological expansion, but after it becomes massive so that we can use Eq. (28). As before, we calculate first the gravitational wave constraint. The curvaton field decays at a time when $\Gamma_\sigma = H$ and then

$$\frac{\Gamma_\sigma}{H_{\text{kin}}} = \frac{a_{\text{kin}}^3}{a_{\text{d}}^3}, \quad (33)$$

where “d” labels the different quantities at the time of curvaton decay. The radiation produced from curvaton decay has an energy density

$$\rho_r^{(\sigma)} = \frac{m^2 \sigma_i^2}{2} \frac{a_{\text{m}}^3}{a_{\text{d}}^3} \frac{a_{\text{d}}^4}{a^4}. \quad (34)$$

We will now assume that this radiation energy density is much larger than that produced by inflation, a necessary requirement if we are to maintain the gravitational waves under control. From Eq. (34), radiation equals the stiff scalar matter ($\rho_r^{(\sigma)} = \rho_\phi$) at a time given by

$$\left. \frac{\rho_r^{(\sigma)}}{\rho_\phi} \right|_{a=a_{\text{eq}}} = \frac{4\pi}{3} \frac{m^2 \sigma_i^2}{m_{\text{Pl}}^2 H_{\text{kin}}^2} \frac{a_{\text{m}}^3}{a_{\text{kin}}^3} \frac{a_{\text{eq}}^2}{a_{\text{kin}}^2} \frac{a_{\text{d}}}{a_{\text{kin}}} = 1. \quad (35)$$

This equation defines the ratio $a_{\text{kin}}/a_{\text{eq}}$ to be used in Eq. (15). Hence, the constraint from gravitational waves now reads

$$\left. \frac{\rho_g}{\rho_r} \right|_{a=a_{\text{eq}}} = \frac{16}{\pi^2} h_{\text{GW}}^2 \sqrt[3]{\frac{\Gamma_\sigma}{H_{\text{kin}}}} \frac{m}{H_{\text{kin}}} \frac{m_{\text{Pl}}^2}{\sigma_i^2} \ll 1, \quad (36)$$

where we have also used Eqs. (24) and (33).

Finally, we derive the new constraints on Γ_σ . The curvaton field should decay after it becomes massive, so that $\Gamma_\sigma < m$; and before it dominates the expansion of the Universe, $\Gamma_\sigma > H_{\text{eq}}$ [see Eq. (31)]. Therefore, the constraints on the decay parameter are

$$\frac{4\pi}{3} \frac{\sigma_i^2}{m_{\text{Pl}}^2} m < \Gamma_\sigma < m. \quad (37)$$

E. The surviving inflaton field

For completeness, we should add a small note here about the evolution of the inflaton field after curvaton domination and/or decay. It is well known that a steep exponential potential has a tracker behavior where it evolves with the same equation of state as the dominant component [19]. However, during the kination regime, the field is fast rolling, and this typically takes it to very low energy densities, so that it can join the tracker only late in the cosmological evolution. Detailed analysis, following Ref. [16], would be needed to determine its precise evolution, but it is unlikely to have any significant cosmological consequences unless the exponential form of the potential becomes modified at low energies.

F. Curvaton fluctuations

Equations (30) and (37) are the constraints on the free parameters of the model coming from gravitational waves, but their actual form is not useful yet because of the term h_{GW}^2 . The latter is indeed related to other parameters of the model through the curvaton fluctuations, as we shall show in this section.

The curvaton fluctuations $\delta\sigma_k$ obey the linearized Klein-Gordon equation

$$\delta\ddot{\sigma}_k + 3H\delta\dot{\sigma}_k + \left[\frac{k^2}{a^2} + m^2 \right] \delta\sigma_k = 0, \quad (38)$$

where k is the comoving wave number.

During the time the fluctuations are inside the horizon, they obey the same differential equation as do the inflaton fluctuations, from which we conclude that they acquire the amplitude $\delta\sigma_i \simeq H_i/2\pi$. Once the fluctuations are out of the horizon, they obey the same differential equation as does the unperturbed curvaton field and then we expect that they remain constant during inflation, under quite general initial conditions.

Up to this point, the evolution of the curvaton fluctuations resembles that of previous scenarios already present in the literature. But, in a similar manner to our analysis of the homogeneous curvaton field in Secs. III C and III D, we should now take into account that the inflaton field survives the inflationary period, and study how the final spectrum of perturbations could be modified by this.

Following previous work, we can calculate the spectrum of the Bardeen parameter \mathcal{P}_ζ , whose observed value is about 2×10^{-9} . When decay occurs *after* curvaton domination, the produced perturbation is [1,2,18]

$$\mathcal{P}_\zeta \approx \frac{1}{9\pi^2} \frac{H_i^2}{\sigma_i^2}. \quad (39)$$

This case is by far the simplest one. The spectrum of fluctuations is automatically Gaussian since $\sigma_i^2 \gg H_i^2/4\pi^2$, and is independent of Γ_σ , a feature that will simplify the analysis of the parameter space. Moreover, even though the curvaton field coexists with the inflaton field up to this epoch, the spectrum of fluctuations is the same as in the standard scenario.

In the limit in which the curvaton decays when subdominant, the Bardeen parameter is given by [1,2]

$$\mathcal{P}_\zeta \approx \frac{r_d^2}{36\pi^2} \frac{H_i^2}{\sigma_i^2}, \quad (40)$$

where the normalization takes into account that the dominant component at that time is the scalar stiff matter. Here r_d is the ratio of curvaton energy density to stiff scalar matter at curvaton decay, which is found by using Eqs. (24) and (33) in

$$r_d = \frac{\rho_\sigma}{\rho_\phi} \bigg|_{a=a_d} = \frac{4\pi}{3} \frac{m^2 \sigma_i^2}{m_{\text{Pl}}^2 H_{\text{kin}}^2} \frac{a_m^3}{a_{\text{kin}}^3} \frac{a_d^3}{a_{\text{kin}}^3} = \frac{4\pi}{3} \frac{m}{\Gamma_\sigma} \frac{\sigma_i^2}{m_{\text{Pl}}^2}. \quad (41)$$

Contrary to the previous case, Eqs. (40) and (41) show explicitly the influence of the surviving inflaton field, and they are markedly different from the standard scenario in which the curvaton field coexists only with radiation.

IV. MODEL CONSTRAINTS

In this section, we use the different constraints on the curvaton field to study under which conditions it can be used together with steep inflation. The two cases that were worked out before will be treated separately.

A. Curvaton decay after domination

In this case, Eqs. (8), (9), and (39) allow us to fix λ_b in terms of σ_i and α as

$$\frac{\lambda_b}{m_{\text{Pl}}^4} = \frac{27\pi}{16} \frac{\mathcal{P}_\zeta}{(\mathcal{N}+1)^2 a^4} \frac{\sigma_i^2}{m_{\text{Pl}}^2}, \quad (42)$$

and the amplitude of primordial gravitational waves Eq. (14) can be written in terms of \mathcal{P}_ζ , σ_i and α as

$$h_{\text{GW}}^2 = 27\pi^2 \alpha^2 (\mathcal{N}+1) \mathcal{P}_\zeta \frac{\sigma_i^2}{m_{\text{Pl}}^2}. \quad (43)$$

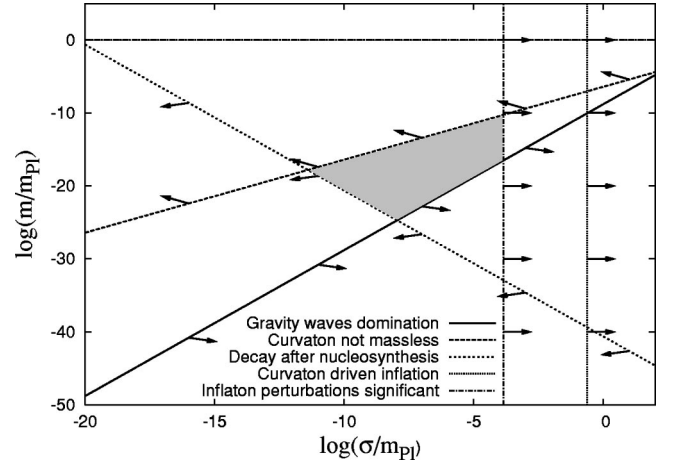


FIG. 1. Allowed region of parameter space of the curvaton–steep inflation model for the case of curvaton domination before decay. Shown are the constraints Eqs. (25), (44), (45), (46) and (47) using $\alpha = 15$ and $\mathcal{N} = 70$. The regions excluded by each constraint are indicated by the arrows, and the allowed region is shaded. The allowed region is reduced for a larger value of α .

Thus, Eqs. (30) and (27) are transformed into the following constraints on the curvaton mass:

$$\frac{m}{m_{\text{Pl}}} \gg 10^3 \sqrt{\pi^3 (\mathcal{N}+1)} \alpha (2.6\alpha^2 - 21) \mathcal{P}_\zeta^2 \frac{\sigma_i^2}{m_{\text{Pl}}^2}, \quad (44)$$

$$\frac{m}{m_{\text{Pl}}} \leq \frac{3\pi \mathcal{P}_\zeta^{1/2}}{\alpha (\mathcal{N}+1)} \frac{\sigma_i}{m_{\text{Pl}}}, \quad (45)$$

where we have used Eq. (13).

The initial value of the curvaton field σ_i is restricted by Eq. (25) and by another restriction coming from the assumption that the inflaton fluctuations are not relevant. The latter is obtained by using Eqs. (11) and (42), which give

$$\frac{\sigma_i}{m_{\text{Pl}}} < \frac{5}{\sqrt{72\pi}} \frac{1}{\alpha (\mathcal{N}+1)} \frac{A_s}{\sqrt{\mathcal{P}_\zeta}}. \quad (46)$$

In other words, Eq. (46) just means that the brane tension has a lower value than that of Eq. (11).

Finally, Eq. (32) restricts the value of the decay parameter Γ_σ , which can be transformed into another constraint upon m and σ_i as

$$\frac{m}{m_{\text{Pl}}} \frac{\sigma_i^2}{m_{\text{Pl}}^2} \gg \frac{3}{4\pi} \times 10^{-40}. \quad (47)$$

The different conditions on the model of steep inflation + curvaton field constrain the values of λ_b , σ_i and m , but not of α and \mathcal{N} which remain free parameters of the model. Therefore, we can draw the allowed region for the former in a plot of m versus σ_i by fixing the value of α . Such a plot is shown in Fig. 1 for $\alpha = 15$ and $\mathcal{N} = 70$, where we can see that there are viable models satisfying all the requirements.

However, the allowed region of the parameter space is reduced for larger values of the self-interaction of the inflaton field $\alpha \gg 1$, as we can see from Eqs. (44) and (45). Thus, it is not always possible to find a successful model of curvaton field + steep inflation, and this is mainly because of the gravitational wave constraint Eq. (36).

B. Curvaton decay before domination

For this case, it is convenient to use Eqs. (40) and (41) to write σ_i in terms of the other quantities as

$$\frac{\sigma_i^2}{m_{\text{Pl}}^2} = \frac{81}{4} \frac{\mathcal{P}_\zeta}{(\mathcal{N}+1)^2} \frac{H_{\text{kin}}^2}{H_f^2} \frac{m_{\text{Pl}}^2}{H_{\text{kin}}^2} \frac{\Gamma_\sigma^2}{m^2}. \quad (48)$$

On the other hand, the amplitude of primordial gravitational waves Eq. (14) is rewritten as

$$h_{\text{GW}}^2 = 3\alpha^2(\mathcal{N}+1)^3 \frac{H_f^2}{H_{\text{kin}}^2} \frac{H_{\text{kin}}^2}{m_{\text{Pl}}^2}, \quad (49)$$

and then the gravitational wave constraint Eq. (36) reads

$$\frac{64\alpha^2}{27\pi^2\mathcal{P}_\zeta}(\mathcal{N}+1)^5 \frac{H_f^4}{H_{\text{kin}}^4} \frac{H_{\text{kin}}^4}{m_{\text{Pl}}^4} \left(\frac{H_{\text{kin}}}{\Gamma_\sigma}\right)^{5/3} \frac{m^3}{H_{\text{kin}}^3} \ll 1. \quad (50)$$

In the same manner, Eq. (37) is now written as

$$\frac{27\pi\mathcal{P}_\zeta}{(\mathcal{N}+1)^2} \frac{H_{\text{kin}}^2}{H_f^2} \frac{m_{\text{Pl}}^2}{H_{\text{kin}}^2} \frac{\Gamma_\sigma^2}{m} < \Gamma_\sigma < m. \quad (51)$$

Notice that, from Eqs. (25) and (51) respectively, there are three similar constraints on the curvaton mass of the form

$$\frac{m}{m_{\text{Pl}}} > \left(\frac{\sqrt{27\pi\mathcal{P}_\zeta}}{\mathcal{N}+1} \frac{H_{\text{kin}}}{H_f} \frac{m_{\text{Pl}}}{H_{\text{kin}}} \right) \frac{\Gamma_\sigma}{m_{\text{Pl}}}, \quad (52)$$

$$\frac{m}{m_{\text{Pl}}} > \left(\frac{\sqrt{27\pi\mathcal{P}_\zeta}}{\mathcal{N}+1} \frac{H_{\text{kin}}}{H_f} \frac{m_{\text{Pl}}}{H_{\text{kin}}} \right)^2 \frac{\Gamma_\sigma}{m_{\text{Pl}}}, \quad (53)$$

$$\frac{m}{m_{\text{Pl}}} > \frac{\Gamma_\sigma}{m_{\text{Pl}}}, \quad (54)$$

while Eqs. (27) and (50) give the following constraints, respectively:

$$\frac{m}{m_{\text{Pl}}} \ll \frac{1}{\alpha} \frac{H_f}{H_{\text{kin}}} \frac{H_{\text{kin}}}{m_{\text{Pl}}}, \quad (55)$$

$$\frac{m}{m_{\text{Pl}}} \ll \left[\frac{27\pi^2\mathcal{P}_\zeta}{64\alpha^2(\mathcal{N}+1)^5} \frac{H_{\text{kin}}^4}{H_f^4} \right]^{1/3} \left(\frac{m_{\text{Pl}}^8}{H_{\text{kin}}^8} \frac{\Gamma_\sigma^5}{m_{\text{Pl}}^5} \right)^{1/9}. \quad (56)$$

At this point, we should take into account the Gaussianity condition upon the curvaton perturbations, since it is not automatically satisfied as in Sec. IV A. The Gaussianity condition implies that the perturbations must be small compared to

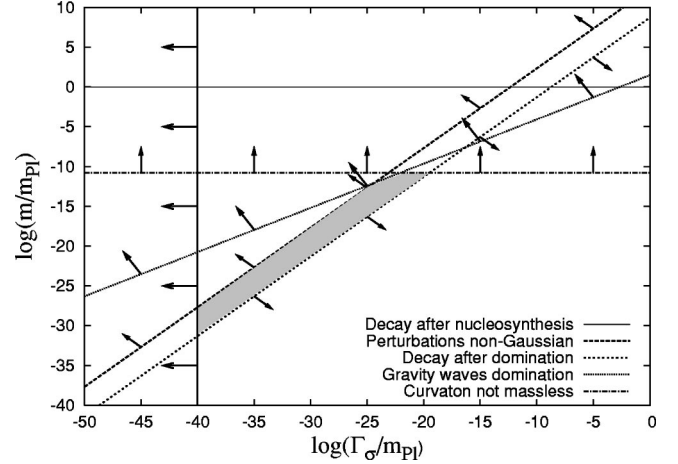


FIG. 2. Curvaton constraints for the case in which the curvaton field decays before domination, using $\alpha=15$, $\mathcal{N}=70$ and $(H_{\text{kin}}/m_{\text{Pl}})=2.13 \times 10^{-11}$ in Eqs. (50), (52)–(57), (58) and the nucleosynthesis constraint. As in Fig. 1, the regions excluded by each constraint are indicated by the arrows, and the allowed region is shaded. The allowed region would be smaller for a smaller (larger) value of H_{kin} (α).

the mean value of the curvaton field, and then $\sigma_i^2 \gg H_i^2/4\pi$. Using Eqs. (8), (13) and (48), we obtain

$$\frac{m}{m_{\text{Pl}}} \ll \frac{9\sqrt{\pi\mathcal{P}_\zeta}}{(\mathcal{N}+1)^2} \frac{H_{\text{kin}}^2}{H_f^2} \frac{m_{\text{Pl}}^2}{H_{\text{kin}}^2} \frac{\Gamma_\sigma}{m_{\text{Pl}}}. \quad (57)$$

All these constraints can be satisfied if $m, \Gamma_\sigma \rightarrow 0$ independently of the value of H_{kin} , but we know that this should not be possible since we still expect to recover the standard big bang scenario at nucleosynthesis. Hence, we will also impose the restriction $\Gamma_\sigma > 10^{-40} m_{\text{Pl}}$.

The final constraint has to do with the inflaton fluctuations, which should be negligible compared to the curvaton ones. In principle, this is a constraint upon λ_b through Eq. (11), but Eqs. (9) and (13) allow us to transform it into a constraint on H_{kin} in the form

$$\frac{H_{\text{kin}}}{m_{\text{Pl}}} \ll \frac{6.21 \times 10^{-5}}{\alpha^3(\mathcal{N}+1)^2} (0.085\alpha^2 - 0.688). \quad (58)$$

To unravel the parameter space allowed by the different constraints, we proceed as follows. We will take as free parameters the inflationary ones, α , \mathcal{N} and H_{kin} , but the latter will take only values permitted by Eq. (58). Once we fix the values of α , \mathcal{N} and H_{kin} by hand, we plot only the strongest constraint of Eqs. (52)–(54) (the other two will be automatically satisfied) and the constraints in Eqs. (55), (56) and (57). An example of this is shown in Fig. 2 for the values $\alpha=15$ and $\mathcal{N}=70$, where we also assumed that inflaton fluctuations contribute as much as the 10% of the Cosmic Background Explorer (COBE) signal.

The different constraints permit some room for viable models, although the allowed region is smaller for smaller values of H_{kin} , i.e., smaller values of λ_b , and for larger

values of α . Thus, we conclude that a realistic scenario in which the curvaton field decays before domination is very restricted by the different constraints.

V. CONCLUSIONS

The steep inflation model is of interest as it ends inflation in a novel manner, through reduction of inflation-sustaining braneworld corrections, and because it generates inflation from the steep exponential potentials often found in supergravity phenomenology. Unfortunately, the original steep inflation model appears unable to match observations, due to an excessive amplitude of short-scale gravitational waves and because its perturbations are not close enough to scale invariance.

In this paper we have shown that curvaton reheating can alleviate these difficulties, by dramatically raising the reheat temperature (as compared to gravitational particle production reheating), while simultaneously suppressing the large-angle gravitational wave contribution. The model is subject to a complex network of constraints, but viable models do exist, provided the brane tension λ_b and the exponential slope α are not too large. We should stress that our analysis applies

only to the case where the inflaton survives until late times, since an early decay of the inflaton field would lead to the standard scenario already studied in the literature [1,2,18].

An interesting feature of the steep inflation model is that the inflaton field may survive to the present, and with suitable modification of the potential from its exponential form at low energies it could act as either dark matter [16,17] or quintessence [8,11,20]. This resembles the scenarios given in Ref. [2] concerning residual isocurvature matter perturbations. For instance, if cold dark matter CDM is created before curvaton decay, then there is a maximal correlation between curvature and CDM-isocurvature perturbations which is at variance with current observations. We expect that the inflaton perturbations will be affected in the same way if the inflaton field is to be the dark matter at late times. The case of quintessence should not be troublesome, since the quintessence perturbations should decay upon horizon entry. However, these situations merit more detailed investigation.

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- [1] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002).
 - [2] D. H. Lyth, C. Ungarelli, and D. Wands, Phys. Rev. D **67**, 023503 (2003).
 - [3] S. Mollerach, Phys. Rev. D **42**, 313 (1990).
 - [4] T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001); Phys. Rev. D **66**, 063501 (2002).
 - [5] K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002).
 - [6] B. Feng and M. Li, Phys. Lett. B **564**, 169 (2003).
 - [7] L. H. Ford, Phys. Rev. D **35**, 2955 (1987); L. P. Grishchuk and Y. V. Sidorov, *ibid.* **42**, 3413 (1990); B. Spokoiny, Phys. Lett. B **315**, 40 (1993).
 - [8] E. J. Copeland, A. R. Liddle, and J. E. Lidsey, Phys. Rev. D **64**, 023509 (2001).
 - [9] M. Joyce and T. Prokopec, Phys. Rev. D **57**, 6022 (1998).
 - [10] V. Sahni, M. Sami, and T. Souradeep, Phys. Rev. D **65**, 023518 (2002).
 - [11] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **59**, 063505 (1999).
 - [12] K. Dimopoulos, astro-ph/0212264.
 - [13] E. I. Guendelman, Mod. Phys. Lett. A **14**, 1043 (1999).
 - [14] R. Maartens, D. Wands, B. A. Bassett, and I. P. C. Heard, Phys. Rev. D **62**, 041301(R) (2000).
 - [15] M. Joyce, Phys. Rev. D **55**, 1875 (1997).
 - [16] P. G. Ferreira and M. Joyce, Phys. Rev. D **58**, 023503 (1998).
 - [17] J. E. Lidsey, T. Matos, and L. A. Ureña-López, Phys. Rev. D **66**, 023514 (2002).
 - [18] N. Bartolo and A. R. Liddle, Phys. Rev. D **65**, 121301(R) (2002).
 - [19] P. J. E. Peebles and B. Ratra, Phys. Rev. D **37**, 3406 (1988); C. Wetterich, Nucl. Phys. B **302**, 668 (1988); E. J. Copeland, A. R. Liddle, and D. Wands, Ann. N.Y. Acad. Sci. **688**, 647 (1993); Phys. Rev. D **57**, 4686 (1998).
 - [20] G. Huey and J. E. Lidsey, Phys. Lett. B **514**, 217 (2001).